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# Reinvestigating the R2 Indicator: Achieving Pareto Compliance by Integration

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## Summary

The R2 indicator is a set-based performance indicator for multi-objective optimization, which describes the expected utility of a solution set under a distribution of utility functions.

In practice, this distribution is always **discrete**, yielding just a **weakly Pareto-compliant indicator**.

We show how to compute the R2 indicator for a **continuous distribution of utility functions** for bi-objective problems efficiently, which yields a **Pareto-compliant indicator**. This is the first known Pareto-compliant indicator which requires **only information on an ideal point**.

## Background

The quality of multi-objective solution sets are assessed using **quality indicators**.

The **hypervolume** indicator is state of the art and measures the dominated hypervolume relative to an anti-optimal reference point. It is Pareto-compliant, but sensitive to the placement of the reference point.

The **R2 indicator** for a solution set $Y$ and a distribution of utility functions $U$ is originally defined by the following integral, though in practice it is only **approximated using a discrete distribution of weights $W$** :

$$R2(Y) := \int_{u \in U} \min_{y \in Y} u(y) du \approx \frac{1}{|W|} \sum_{w \in W} \min_{y \in Y} u_w(y)$$

The most common choice of utilities are **Tchebycheff** aggregation functions:

$$u_w(y) = \max_{i=1,\dots,m} w_i(y_i - y_i^*) = \max_{i=1,\dots,m} w_i y_i'$$

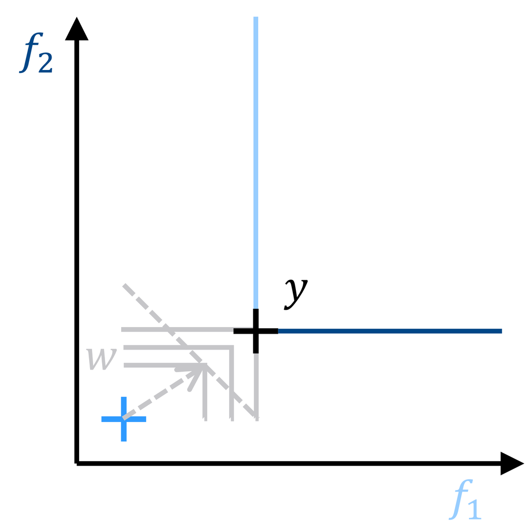
where $\sum_i w_i = 1$ and all $w_i \geq 0$.

## Exact R2 Computation

### R2 for a Single Solution

We need to find the change of maximiser from the first to the second term at $w^* = \frac{y_2}{y_1 + y_2}$:

$$R2(Y) = \int_0^1 \max(y_1 w, y_2(1-w))\, dw$$
$$= \int_0^{w^*} y_2(1-w) dw + \int_{w^*}^1 y_1 w\, dw$$
$$= \frac{1}{2} y_2 (1 - (1-w^*)^2) + \frac{1}{2} y_1 (1 - (w^*)^2)$$



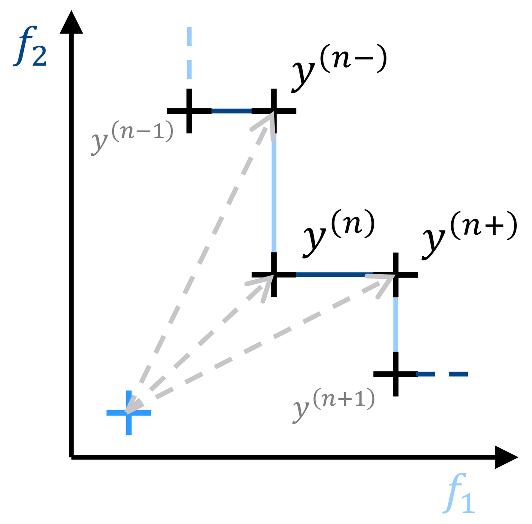

### R2 for a Set of Solutions

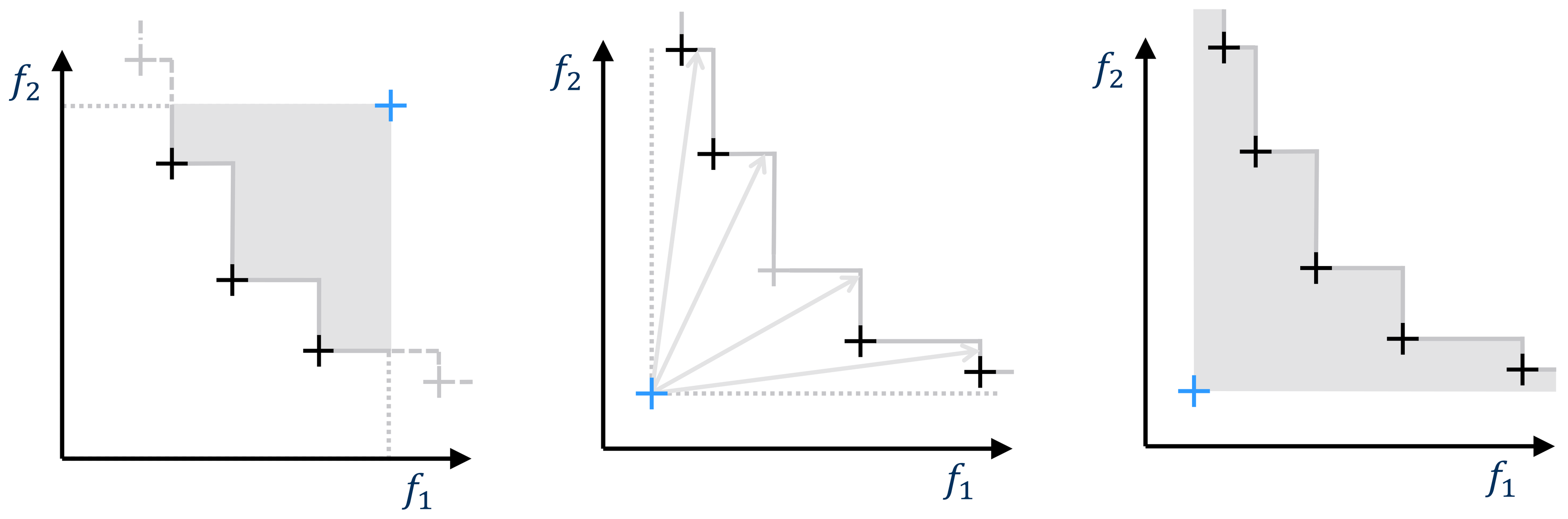
Split up the computation by the weights for which each solution in a set is optimal:

$$R2(y^{(n)}) = \frac{1}{2} y_2^{(n)} \left( \left(1 - w_1^{(n-)}\right)^2 - \left(1 - w_1^{(n)}\right)^2 \right) + \frac{1}{2} y_1^{(n)} \left( \left(w_1^{(n+)}\right)^2 - \left(w_1^{(n)}\right)^2 \right)$$

The exact R2 value is the sum of the individual values:

$$R2(Y) = \sum_{n=1}^{N} R2(y^{(n)})$$




**HV Indicator**
- \+ Pareto-compliant in dominating area
- \+ Intuitive geometric interpretation
- – Requires anti-optimal reference point
- – Can only assess dominating points

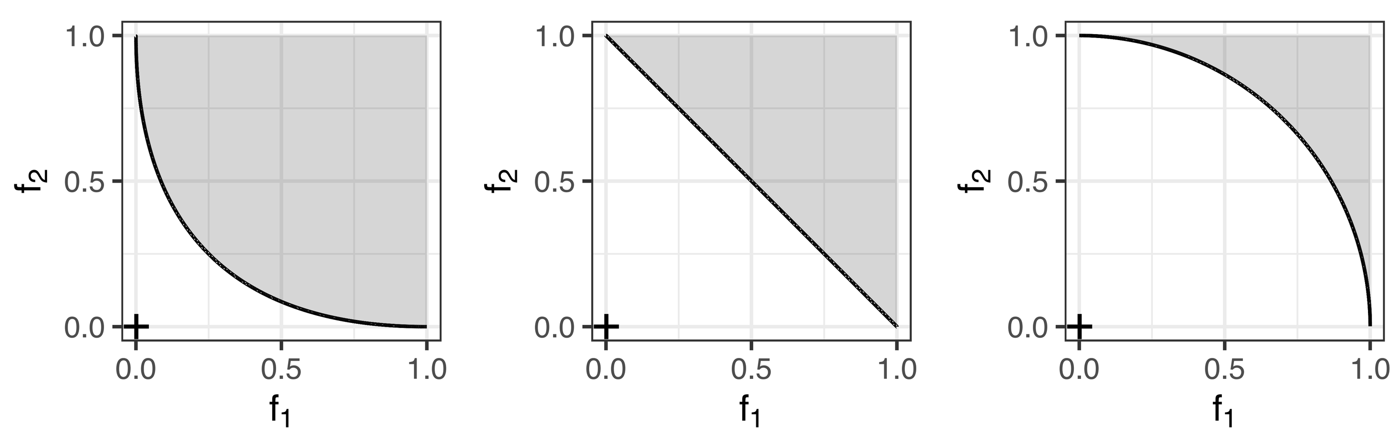
**Discrete R2 Indicator**
- \+ Only requires ideal point
- – May miss some points in evaluation
- – Only weakly Pareto-compliant

**Exact R2 Indicator**
- \+ Only requires ideal point
- \+ Fully Pareto-compliant
- – No intuitive geometric interpretation

## Properties

In normalized objective space, the R2 indicator has the following special values:

**Ideal** point: $R2(\{0,0\}) = 0$ and **Nadir** point: $R2(\{1,1\}) = \frac{3}{4}$




convex: $0 < R2 < \frac{1}{6}$ linear: $R2 = \frac{1}{6}$ concave: $\frac{1}{6} < R2 < \frac{1}{4}$

**Value ranges for R2 in normalized objective space.**

## Outlook

The exact R2 indicator is a **great alternative to hypervolume-based assessment**, especially if an ideal reference point is naturally available.

Integration into **benchmarking**, **algorithm design**, as well as better **understanding of the discrete R2 indicator** are promising next steps.

**Higher-dimensional extensions** are already being studied.

## More About R2

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**Code** **Paper** **Poster**

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