Hot off the Press: BONO-Bench

A Comprehensive Test Suite for Bi-objective Numerical Optimization with Traceable Pareto Sets

Lennart Schäpermeier, *Computational Social Science and Systems Analysis*, *University of Münster*
Pascal Kerschke, *Big Data Analytics in Transportation*, *TU Dresden* & *ScaDS.AI Dresden/Leipzig*

Summary

- Benchmarking is a fundamental aspect of research in numerical multi-objective black-box optimization.
- Most wide-spread test problems are either (1) hand-constructed with well-understood solutions but unrealistic problem properties or (2) composited from complex single-objective problems with little insight into Pareto-optimal solutions.
- BONO-Bench is a **scalable generator and benchmark set for bi-objective problems with tunable problem properties** built from convex-quadratic multiple peaks problems. At the same time, it has known reference solutions, making it ideal for benchmarking.

Numerical Bi-objective Optimization

A multi-objective black-box optimization problems with k objectives in a real-valued decision space $\mathcal{X} \subseteq R^d$ is defined as:

$$\underset{x \in \mathcal{X}}{\text{minimize}} \quad F(x) = (f_1(x), \ldots, f_k(x)), \qquad f_i(x) : \mathcal{X} \to \mathbb{R}.$$

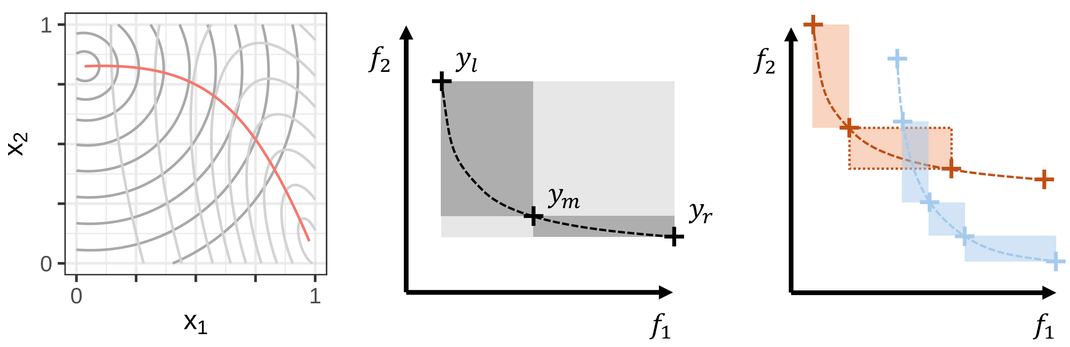
In particular, we consider **bi-objective** problems ($k = 2$) with **box constraints** $\mathcal{X} = [-5, 5]^d$. As the solution, the set of optimal trade-offs $\mathcal{X}^*$ (i.e., the Pareto set) is sought. $\mathcal{X}^*$ is usually approximated by means of heuristics optimization approaches like evolutionary algorithms or Bayesian optimization, requiring structured evaluation on well-known test problems to assess algorithm performance.

Bi-objective Convex-quadratic Problems

All objectives in BONO-Bench are based on a multi-peak function:

$$f_i(x) = s \cdot \left(\min_{j \in \{1,\ldots,J\}} \left\{ \frac{1}{2}(x - x_j^*)^T H_j (x - x_j^*) + y_j^* \right\} \right)^{p/2} + y^*.$$

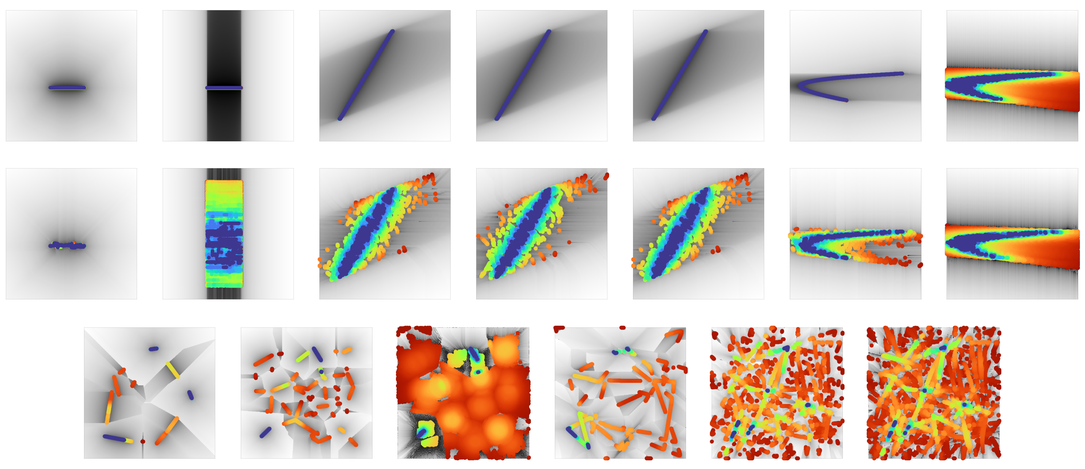
Pareto sets of two convex-quadratic functions are analytically tractable and can be numerically evaluated to an arbitrary degree, even under monotone transformations. This allows us to compute precise reference solutions for all BONO-Bench problems.



For all BONO-Bench problems, we perform approximations of the Pareto front up to a target precision for the hypervolume and exact R2 quality indicators. These reference solutions enable precise performance assessment using runtime profiles.

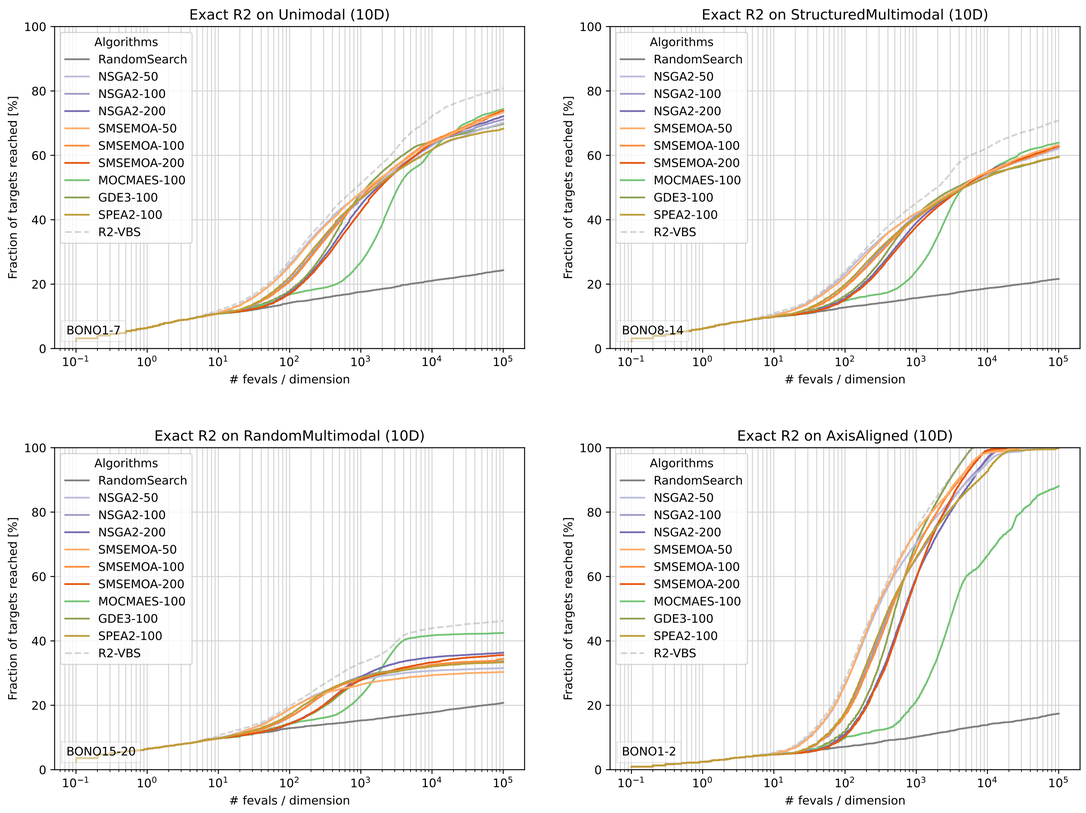
BONO-Bench Problem Set

The BONO-Bench problem set contains 20 problems, grouped into unimodal (BONO1-7), multimodal with global structure (BONO8-14) and without global structure (BONO15-20). Decision space (PLOT) visualizations for the first instance (out of 15) for each 2D BONO-Bench problem:



Performance Assessment via Runtime Profiles

Performance assessment utilizes runtime profiles, aggregating the solution progress according to the chosen quality indicator. Runtime profiles can be analyzed per problem, or aggregated to focus analysis on a particular problem property:



Future Research

- We aim to build an **extensive library of benchmarked algorithms**.
- We want to **extend the problem properties** supported by the BONO-Bench generator.
- BONO-Bench offers a great playground for **algorithm selection** and configuration of **multi-objective optimization heuristics**.

References

Schäpermeier, L. & Kerschke, P. (2026). BONO-Bench: A Comprehensive Test Suite for Bi-objective Numerical Optimization with Traceable Pareto Sets. Accepted in ACM Transactions on Evolutionary Learning and Optimization. DOI: 10.1145/3795775.

GitHub: `schaepermeier/bonobench`